Introduction to compilers and interpreters

基础软件理论与实践公开课

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Logistics

- Course website: https://bobzhang.github.io/courses/
- Discussion forum: https://bbs.csdn.net/forums/raelidea
- Target audience:
  - People who are interested in language design and implementations
  - No PL theory pre-requisites
- Example code language: ReScript
  - Homebrew
  - ReScript is a dialect of ML: Why ML are good for writing compilers
  - Easy to install on major platforms including Windows

We are hiring

- 地点：深圳
- 程序语言工具链， 开发者工具，垃圾回收，代码编辑器，IDE等

Introduction

Why study compiler&interpreters ?

- It is fun
- Understand your tools you use everyday
- Understand the cost of abstraction
  - Hidden allocation when declaring local functions
  - Why memory leak happens
- Make your own DSLs for profit
- Develop a good taste

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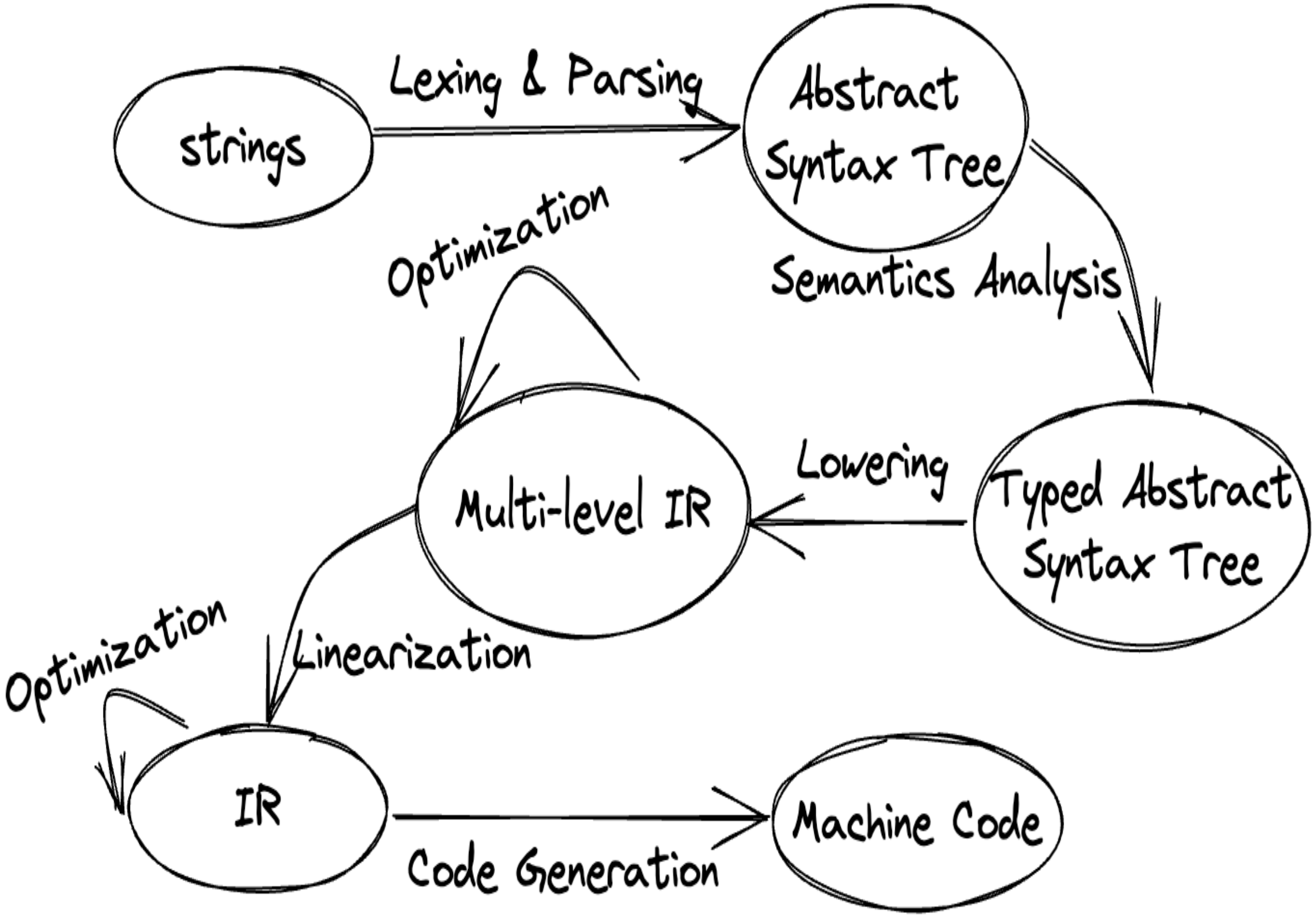
Course Overview

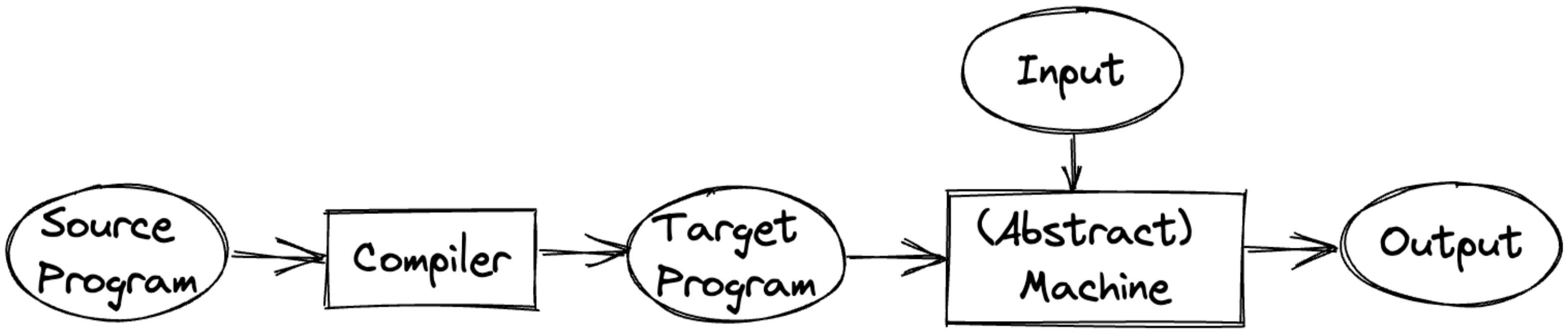
Lec	Topic	Lec	Topic
0	Introduction	6	Stack machine and compilation
1	ReScript crash course	7	WebAssembly
2	λ Calculus	8	Garbage Collection and Memory Management
3	Names, Binders, De Bruijn index	9	Type checking
4	Closure Calculus	10	Type Inference and Unification
5	Pattern Matching	11&12	Formal Verification, Guest Lectures

Compilation Phases

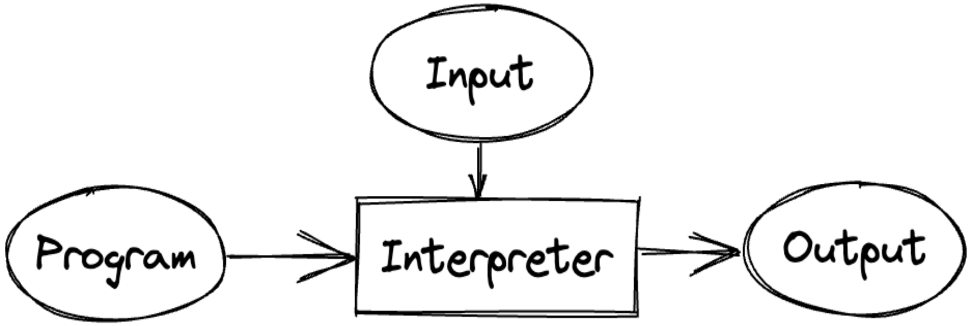


Compilers, Transpilers, Interpreters

- Compilation and execution in two stages



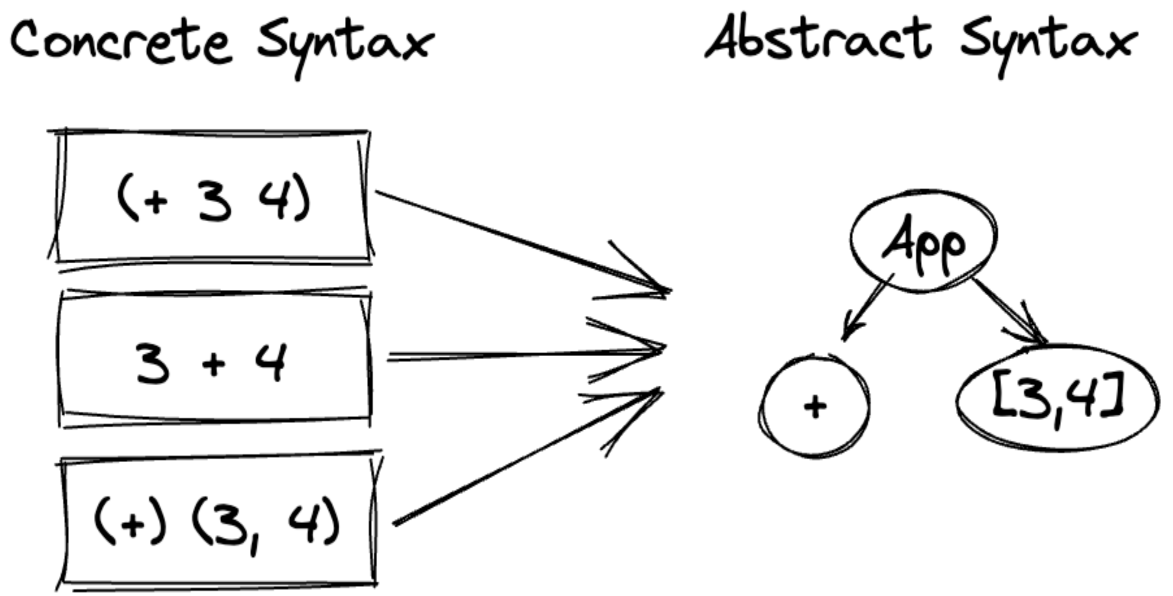
- Interpretation in one stage



- Transpilers translate a source code from a language to another at similar level of abstraction

Lexing & Parsing

- From strings to an abstract syntax tree
- Usually split into two phases: tokenization and parsing
- Lots of tool support, e.g.
  - Lex, Yacc, Bison, Menhir, Antlr, TreeSitter, parsing combinators, etc.



Semantic Analysis

- Build the symbol table, resolve variables, modules
- Type checking & inference
    - Check that operations are given values of the right types
    - Infer types when annotation is missing
    - Typeclass/Implicits resolving
    - check other safety/security problems
        - Lifetime analysis
- Type soundness: no runtime type error when type checks

Language specific lowering, optimizations

- Class/Module/objects/typeclass desugaring
- Pattern match desugaring
- Closure conversion
- Language specific optimizations
- IR relatively rich, MLIR

Linearization & optimizations

- Language & platform agnostics
- Opimizations
  - Constant folding, propogation, CSE, parital evaluation etc
  - Loop invariant code motion
  - Tail call eliminations
  - Intra-procedural, inter-procedural optimization
- IR simplified: three address code, LLVM IR etc

Platform specific code generation

- Instuction selection
- Register allocation
- Instruction scheduling and machine-specific optimization
- Most influential in numeric compuations, DSA

Abstract Syntax vs. Concrete Syntax

- Modern language design: no semantic analysis during parsing
  - Counter example: C++ parsing is hard, error message is cryptic
- Many-to-one relation from concrete syntax to abstract syntax
- Start from abstract syntax for this course
  - Tutorials later for parsing in ReScript

Tiny Language 0

Abstract Syntax

```
type rec expr =
  | Cst (int) // i
  | Add (expr,expr) // a + b
  | Mul (expr,expr) // a * b
```

Interpreter

```
let rec eval = (expr) => {
  switch expr {
  | Cst (i) => i
  | Add(a,b) => eval (a) + eval (b)
  | Mul(a,b) => eval (a) * eval (b)
  }
}
```

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Formalization

Semantics

The evaluation result is a value, which is an integer for our expression language

$$\begin{aligned} \textsf{terms :} \quad & e ::= \textsf{Cst}(i) \mid \textsf{Add}(e_1, e_2) \mid \textsf{Mul}(e_1, e_2) \\ \textsf{values :} \quad & v ::= i \in \textsf{Int32} \end{aligned}$$

The evaluation rules:

$$\frac{}{\textsf{Cst}(i) \Downarrow i}\text{E-const} \qquad \frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2}{\textsf{Add}(e_1, e_2) \Downarrow (v_1 + v_2)}\text{E-add} \qquad \frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2}{\textsf{Mul}(e_1, e_2) \Downarrow (v_1 * v_2)}\text{E-mul}$$

Inference rules

- The evaluation relation $e \Downarrow v$ means expression e evaluates to value v, for example
  - $\mathsf{Cst}(42) \Downarrow 42$
  - $\mathsf{Add}(\mathsf{Cst}(3), \mathsf{Cst}(4)) \Downarrow 7$
- Inference rules provide a concise way of specifying language properties, analyses, etc
  - If the **premises** are true, then the **conclusion** is true
  - An **axiom** is a rule with no premises
  - Inference rules can be **instantiated** by replacing **metavariables** $(e, e_1, e_2, x, i, \cdots)$ with expressions, program variables, integers

Proof Tree

- Instantiated rules can be combined into proof trees
- $e \Downarrow v$ holds if and only if there is a finite proof tree constructed from correctly instantiated rules, and leaves of the tree are axioms

What is the problem of our interpreter?

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Lowering to a stack machine and interpret

```
type instr =  Cst (int) | Add | Mul
```

```
let rec eval = (instrs,stk) => {
  switch (instrs,stk) {
  | (list{ Cst (i), ... rest},_) =>
         eval(rest, list{i,...stk})
  | (list{Add, ... rest}, list{a,b,...stk}) =>
         eval(rest, list{a+b, ...stk})
  | (list{Mul, ... rest}, list{a,b,...stk}) =>
         eval(rest, list{a*b, ...stk})
  | _ => assert false
  }
}
```

Semantics

The machine has two components:

- a code pointer c giving the next instruction to execute
- a stack s holding intermediate results

Notation for stack: top of stack is on the left

$$
\begin{aligned}
s &\to v :: s && (\text{push } v \text{ on } s) \\
v :: s &\to s && (\text{pop } v \text{ off } s)
\end{aligned}
$$

Transition of Stack Machine

Code and stack:

$$\begin{array}{ll} \text{code :} & c ::= \epsilon \mid i \,;\, c \\ \text{stack :} & s ::= \epsilon \mid v :: s \end{array}$$

Transition of the machine:

$$\begin{array}{rcll} (\mathrm{Cst}(i); c, s) & \to & (c, s) & (\text{I-Cst}) \\ (\mathrm{Add}; c, n_2 :: n_1 :: s) & \to & (c, (n_1 + n_2) :: s) & (\text{I-Add}) \\ (\mathrm{Mul}; c, n_2 :: n_1 :: s) & \to & (c, (n_1 \times n_2) :: s) & (\text{I-Mul}) \end{array}$$

The execution of a sequence of instructions terminates when the code pointer reaches the end and returns the value on the top of the stack

$$\frac{(c, \epsilon) \to^* (\epsilon, v :: s)}{c \downarrow v}$$

Formalization

The compilation corresponds to the following mathematical formalization.

$$
\begin{aligned}
[\![\mathsf{Cst}(i)]\!] &= \mathrm{Cst}(i) \\
[\![\mathsf{Add}(\mathsf{e}_1, \mathsf{e}_2)]\!] &= [\![e_1]\!] \; ; \; [\![e_2]\!] \; ; \; \mathrm{Add} \\
[\![\mathsf{Mul}(\mathsf{e}_1, \mathsf{e}_2)]\!] &= [\![e_1]\!] \; ; \; [\![e_2]\!] \; ; \; \mathrm{Mul}
\end{aligned}
$$

- $[\![\cdots]\!]$ is a commonly used notation for compilation
- Invariant: stack balanced property
- Proof by induction (machine checked proof using Coq)

Compilation

- The evaluation **expr** language implicitly uses the stack of the host language
- The stack machine manipulates the stack explicitly

Correctness of Compilation

A correct implementation of the compiler preserves the semantics in the following sense

$$e \Downarrow v \Longleftrightarrow [\![e]\!] \downarrow v$$

Tiny Language 1

Abstract Syntax: add names

```
type rec expr =
  ...
  | Var (string)
  | Let (string , expr , expr)
```

Interpreter

Semantics with Environment

```
type env = list<(string, int)>

let rec eval = (expr, env) => {
  switch expr {
  | Cst (i) => i
  | Add(a,b) => eval (a, env) + eval (b, env)
  | Mul(a,b) => eval (a, env) * eval (b, env)
  | Var(x) => assoc (x, env)
  | Let(x,e1,e2) => eval(e2, list{(x,eval(e1,env)), ...env})
  }
}
```

Formalization

$$\text{terms}: \quad e ::= \mathsf{Cst}(i) \mid \mathsf{Add}(e_1, e_2) \mid \mathsf{Mul}(e_1, e_2) \mid \mathsf{Var}(i) \mid \mathsf{Let}(x, e_1, e_2)$$

$$\text{envs}: \quad \Gamma ::= \epsilon \mid (x, v) :: s$$

Notations for the environment:

$$\text{variable access: } \Gamma[x] \qquad \text{variable update: } \Gamma[x := v]$$

The evaluation rules:

$$\frac{}{\Gamma \vdash \mathsf{Cst}(i) \Downarrow i}\text{E-const} \qquad \frac{\Gamma \vdash e_1 \Downarrow v_1 \qquad \Gamma \vdash e_2 \Downarrow v_2}{\Gamma \vdash \mathsf{Add}(e_1, e_2) \Downarrow (v_1 + v_2)}\text{E-add} \qquad \frac{\Gamma \vdash e_1 \Downarrow v_1 \qquad \Gamma \vdash e_2 \Downarrow v_2}{\Gamma \vdash \mathsf{Mul}(e_1, e_2) \Downarrow (v_1 * v_2)}\text{E-mul}$$

$$\frac{\Gamma[x] = v}{\Gamma \vdash \mathsf{Var}(x) \Downarrow v}\text{E-var} \qquad \frac{\Gamma \vdash e_1 \Downarrow v_1 \qquad \Gamma[x := v_1] \vdash e_2 \Downarrow v}{\Gamma \vdash \mathsf{Let}(x, e_1, e_2) \Downarrow v}\text{E-let}$$

What's the problem in our evaluator

- Where is the redundant work and can be resolved in compile time?
- The length of variable name affect our runtime performance!!

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Tiny Language 2

The position of a variable in the list is its binding depth (index)

```
module Nameless = {
  type rec expr =
    ...
    | Var (int)
    | Let (expr, expr)
}
```

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Semantics

Evaluation function

```
type env = list<int>

let rec eval = (Nameless.expr, env) => {
  switch expr {
    | Cst(i) => i
    | Add(a,b) => eval (a, env) + eval (b, env)
    | Mul(a,b) => eval (a, env) * eval (b, env)
    | Var(n) => List.nth (env, n)
    | Let(e1,e2) => eval(e2, list{eval(e1,env), ...env})
  }
}
```

Semantics

Terms and values are the same.

Environments become sequence of values $v_1 :: v_2 :: \cdots :: \epsilon$, accessed by position $s[n]$

$$\textsf{envs}: \qquad s ::= \epsilon \mid v :: s$$

Evaluation rules:

$$\frac{}{s \vdash \mathsf{Cst}(i) \Downarrow i}\text{E-const} \qquad \frac{\Gamma \vdash e_1 \Downarrow v_1 \qquad \Gamma \vdash e_2 \Downarrow v_2}{\Gamma \vdash \mathsf{Add}(e_1, e_2) \Downarrow (v_1 + v_2)}\text{E-add} \qquad \frac{\Gamma \vdash e_1 \Downarrow v_1 \qquad \Gamma \vdash e_2 \Downarrow v_2}{\Gamma \vdash \mathsf{Mul}(e_1, e_2) \Downarrow (v_1 * v_2)}\text{E-mul}$$

$$\frac{s[i] = v}{s \vdash \mathsf{Var}(i) \Downarrow v}\text{E-var} \qquad \frac{s \vdash e_1 \Downarrow v_1 \qquad v_1 \cdot L \vdash e_2 \Downarrow v}{s \vdash \mathsf{Let}(x, e_1, e_2) \Downarrow v}\text{E-let}$$

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Explanation

- The evaluation environment Γ for expr contains both names and values
- The evaluation environment s for Nameless. expr only contains the values, indexes resolved at compile time

Lowering expr to Nameless. expr

```
type cenv = list<string>

let rec comp = (expr : expr , cenv : cenv): Nameless.expr => {
    switch expr {
        | Cst(i) => Cst(i)
        | Add(a,b) => Add(comp(a, cenv), comp(b, cenv))
        | Mul(a,b) => Mul(comp(a, cenv), comp(b, cenv))
        | Var(x) => Var(index(cenv, x))
        | Let(x,e1,e2) => Let(comp(e1, cenv), comp(e2, list{x,...cenv}))
    }
}
```

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Compile Nameless. expr

```
type instr =  ... | Var (int) | Pop | Swap
```

Semantics of the new instructions

$$
\begin{aligned}
(\mathrm{Var}(i); c, s) &\to (c, s[i] :: s) && (\text{I-Var}) \\
(\mathrm{Pop}; c, n :: s) &\to (c, s) && (\text{I-Pop}) \\
(\mathrm{Swap}; c, n_2 :: n_1 :: s) &\to (c, n_2 :: n_1 :: s) && (\text{I-Swap})
\end{aligned}
$$

where $s[i]$ reads the i-th value from the top of the stack

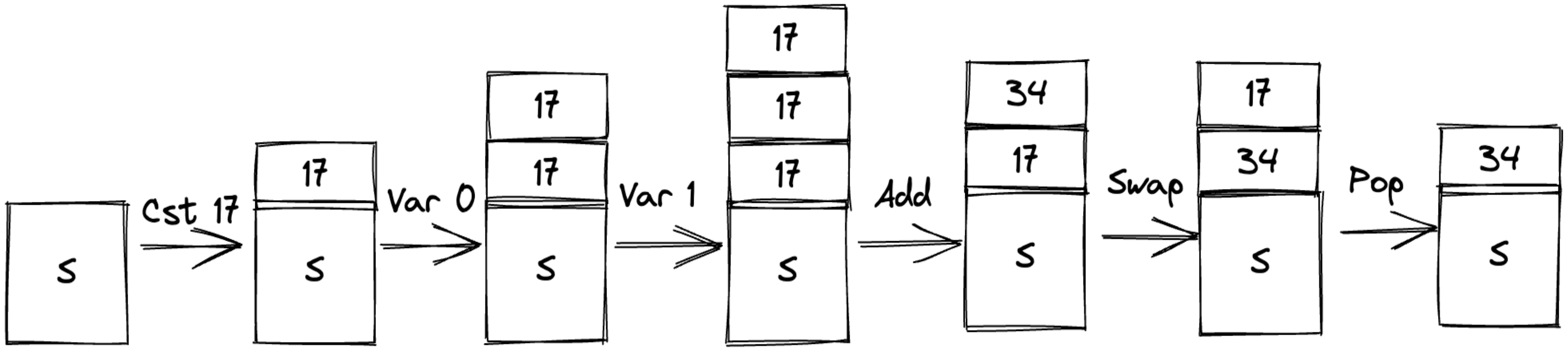
Stack Machine with Variables

The program: $\text{Let}(x, \text{CstI}(17), \text{Add}(\text{Var}(x), \text{Var}(x)))$

is compiled to instructions:

$$[\text{Cst}(17); \text{Var}(0); \color{red}{\text{Var}(1)}; \text{Add}; \text{Swap}; \text{Pop}]$$

The execution on the stack:



Question

- It is obvious we need the $\mathsf{Var}(n)$ instruction to reference variables on the stack
- But why do we need the Swap and Pop instructions?

More Example

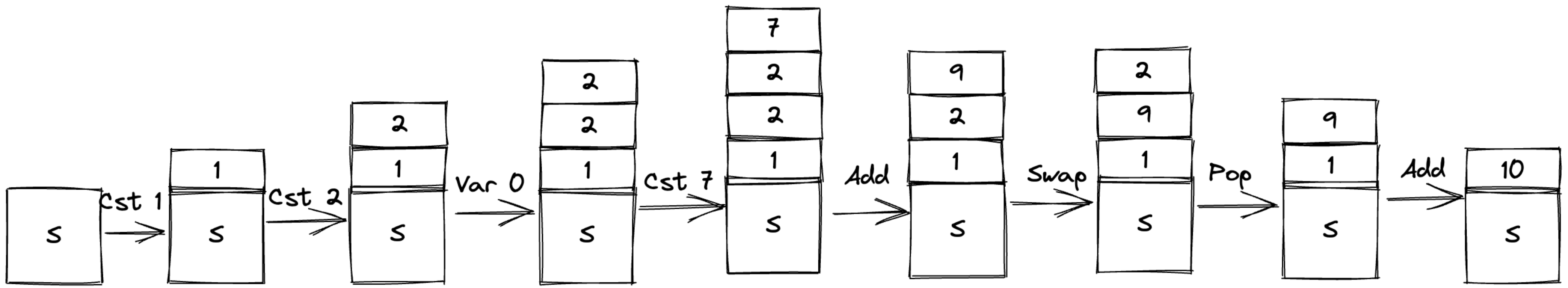
Consider the following program

```
1 + (let x = 2 in x + 7 end)
```

is compiled to instructions

$$[\mathsf{Cst}(1); \mathsf{Cst}(2); \mathsf{Var}(0); \mathsf{Cst}(7); \mathsf{Add}; \mathsf{Swap}; \mathsf{Pop}; \mathsf{Add}]$$

The execution on the stack:

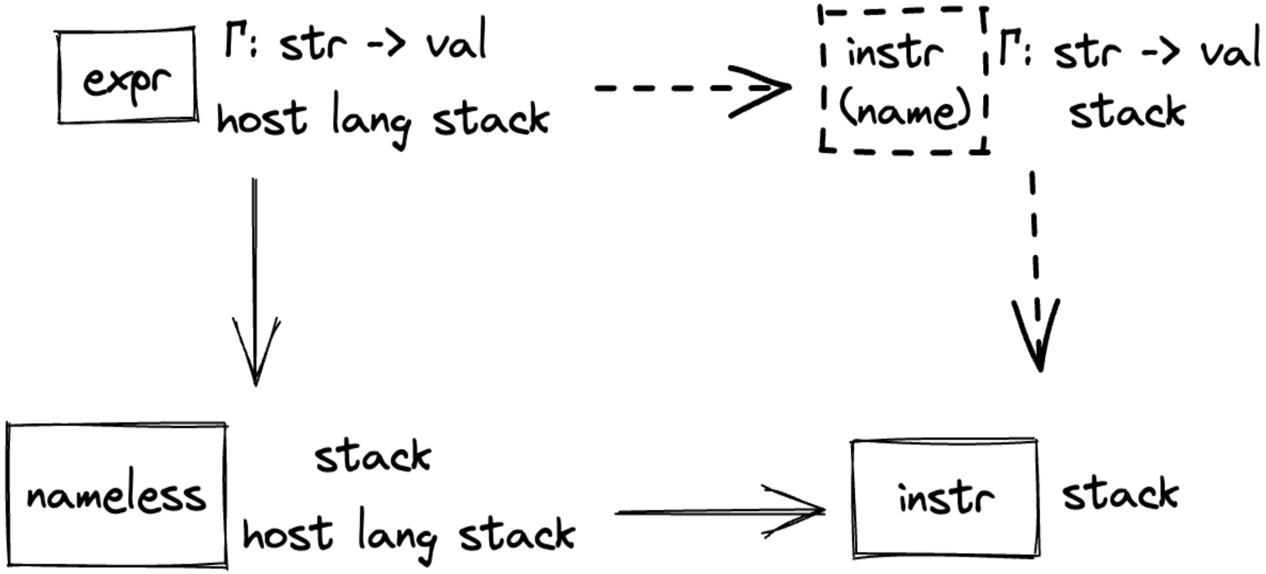


Summary

What have we achieved through compilation? Compare the runtime environment

- Evaluating expr
  - a symbolic environment Γ for local variables
  - (implicit) stack of the host language for temperaries
- Evaluating Nameless. expr
  - a stack for local variables
  - (implicit) stack of the host language for temperaries
- For stack machine instructions, we have
  - a stack for both local variables and temperaries

Summary



Homework

- Write an interpreter for the stack machine with variables
- Write a compiler to translate Nameless. expr to stack machine instructions
- Implement the dashed part (one language + two compilers)