

学院\_\_\_\_\_专业\_\_\_\_\_班 姓名\_\_\_\_\_学号\_\_\_\_\_

## 1.2 数列的极限

### 一、填空题

观察下列数列变化趋势，写出极限.

$$(1) x_n = \frac{1}{2^n} \longrightarrow \lim_{n \rightarrow \infty} x_n = \underline{\hspace{2cm}}.$$

$$(2) x_n = (-1)^n \frac{1}{n} \longrightarrow \lim_{n \rightarrow \infty} x_n = \underline{\hspace{2cm}}.$$

$$(3) x_n = 2 + \frac{1}{n^2} \longrightarrow \lim_{n \rightarrow \infty} x_n = \underline{\hspace{2cm}}.$$

$$(4) x_n = \frac{n-1}{n+1} \longrightarrow \lim_{n \rightarrow \infty} x_n = \underline{\hspace{2cm}}.$$

$$(5) x_n = (-1)^n n \longrightarrow \lim_{n \rightarrow \infty} x_n = \underline{\hspace{2cm}}.$$

$$(6) x_n = \ln \frac{1}{n} \longrightarrow \lim_{n \rightarrow \infty} x_n = \underline{\hspace{2cm}}.$$

### 二、证明题

1. 利用数列极限的定义证明:  $\lim_{n \rightarrow \infty} \frac{2n^2}{n^2 + 1} = 2.$

2. 利用数列极限的定义证明:  $\lim_{n \rightarrow \infty} \frac{\cos(2n)}{n+1} = 0.$

3. 若  $\lim_{n \rightarrow \infty} a_n = a$ , 证明  $\lim_{n \rightarrow \infty} |a_n| = |a|$ , 并举例说明反过来未必成立.

4. 证明: 若  $\lim_{k \rightarrow \infty} x_{2k} = \lim_{k \rightarrow \infty} x_{2k+1} = a$ , 则  $\lim_{n \rightarrow \infty} x_n = a$ .